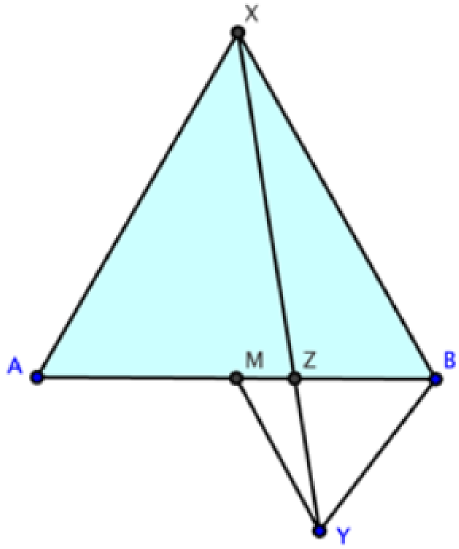
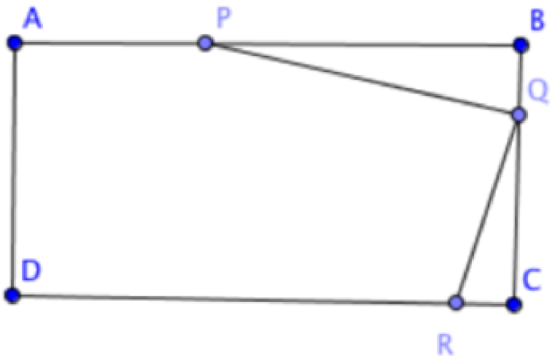
1



In $\triangle XAZ$ and $\triangle YBZ$

$$\angle XZA = \angle YZB, \text{(vertically opposite)}$$
$$\angle XAZ = \angle YBZ = 60^\circ$$
$$\triangle XAB \text{ and} \triangle BMY \text{ are equilateral - given)}$$
$$\therefore \triangle XAZ \sim \triangle YBZ, \text{(AAA)}$$
$$XA = 2YB \text{ (given)}$$
$$\therefore AZ = 2ZB$$

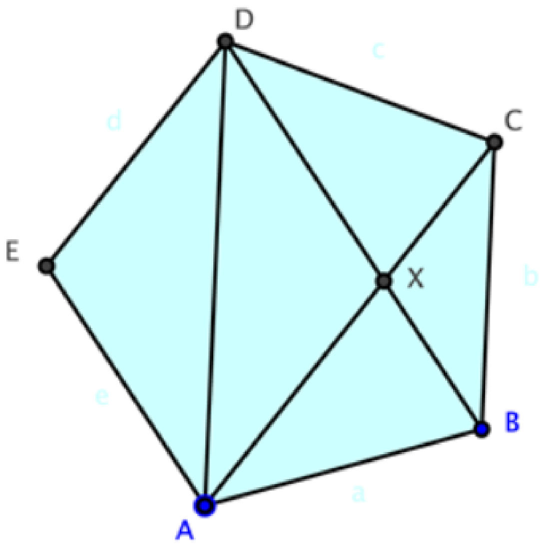
2



In $\triangle PBQ$ and $\triangle QCR$

$$\angle PBQ = \angle QCR, \text{(right angles)}$$
$$\angle PQR = 90^\circ, \text{(given)}$$
$$\therefore \angle RQC = 90^\circ - \angle PQR = \angle BPQ$$
$$\therefore \triangle PBQ \sim \triangle QCR, \text{(AAA)}$$
$$\therefore \frac{PB}{QC} = \frac{BQ}{CR}$$
$$\therefore PB \times CR = BQ \times QC$$

3

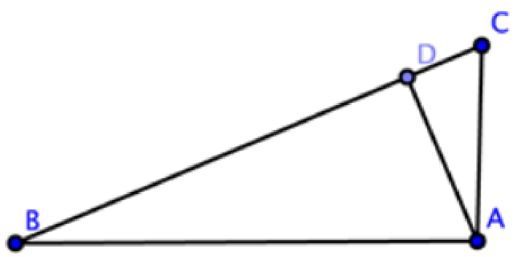


a Interior angles of a regular pentagon are each $108°$
In $\triangle DEF$
$\angle DEF = 108°$ and
$\angle EDA = \angle EAD = 36°$($\triangle DEF$ is isosceles)
Similarly $\angle BAC = 36°$
$\therefore \angle CAE = 72°$

b AC and BD meet at X. In $\triangle BXA$ and $\triangle BAD$

$$\angle XAB = 36° = \angle BDA$$
$$\angle XBA = 72° = \angle ABD$$
$$\therefore \triangle BXA \sim \triangle BAD \text{ (AAA)}$$
$$\therefore \frac{BX}{AB} = \frac{BD}{AB}$$
$$\therefore BX \times BD = AB^2$$

4



a
$$\triangle BAD \sim \triangle BCA \text{ (AAA)} \cdots (1)$$
$$\text{From}(1)$$
$$\frac{AD}{AC} = \frac{AB}{BC}$$
$$\therefore AD \times BC = AB \times AC$$

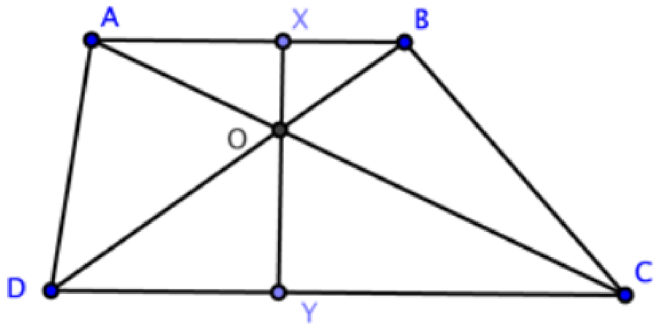
b
$$\triangle BAD \sim \triangle ACD \text{ (AAA)} \cdots (2)$$
$$\therefore \frac{DA}{DC} = \frac{DB}{DA}$$
$$\therefore DA^2 = DC \times DB$$

c $\triangle BAD \sim \triangle BCA\ (\text{AAA}) \cdots (1)$

$$\therefore \frac{BA}{BC} = \frac{DB}{BA}$$

$$\therefore BA^2 = BC \times BD$$

5



$$\triangle AXO \sim \triangle CYO \qquad (AAA) \ldots (1)$$

$$\triangle AOB \sim \triangle COD \qquad (\text{AAA}) \ldots (2)$$

From (1)

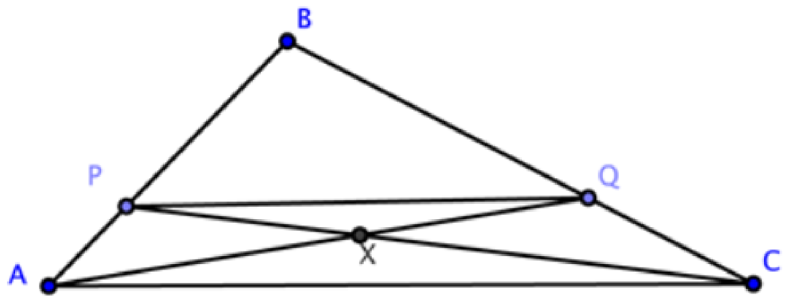
$$\frac{OX}{OY} = \frac{OA}{OC} = \frac{AX}{CY}$$

From (2)

$$\frac{OA}{OC} = \frac{AB}{CD}$$

$$\therefore \frac{OX}{OY} = \frac{OA}{OC} = \frac{AB}{CD}$$

6



$$\triangle PBQ \sim \triangle ABC$$

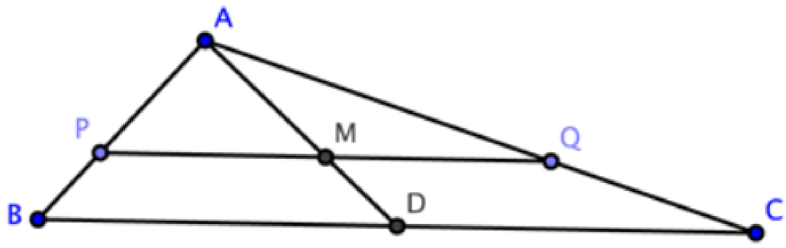
$$\frac{PQ}{AC} = \frac{2}{3}$$

$$\triangle AXC \sim \triangle PXQ$$

$$\frac{PQ}{AC} = \frac{XQ}{XA} = \frac{2}{3}$$

$$\therefore AX : AQ = 3 : 5$$

7



$\triangle APM \sim \triangle ABD \qquad \text{(AAA)}$
$\triangle AMQ \sim \triangle ADC \qquad \text{(AAA)}$

$$\frac{PM}{BD} = \frac{AM}{AD}$$

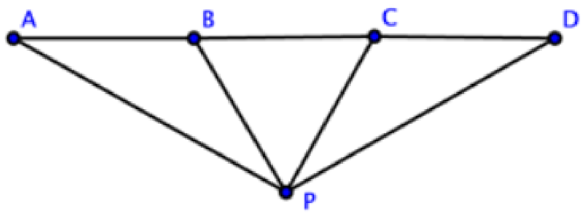
$$\frac{MQ}{DC} = \frac{AM}{AD}$$

$$\therefore \frac{PM}{BD} = \frac{MQ}{DC}$$

Also $BD = DC$

$\therefore PM = MQ$

8



Since $\triangle PBC$ is equilateral:

$\angle PBC = \angle PCB = \angle BPC = 60^\circ$

$\angle PCB = \angle PBC = 120^\circ$

$\triangle PCB$ and $\triangle PAB$ are isosceles.

$\therefore \angle CPD = \angle CDP = 30^\circ$

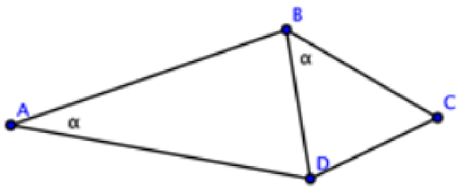
$\angle PAB = \angle BPA = 30^\circ$

$\therefore \triangle APD \sim \triangle ABP \sim \triangle DCP$

$$\therefore \frac{AP}{PB} = \frac{AD}{AP}$$

$\therefore AP^2 = AB \times AD$

9



$\angle BAD = \angle DBC$

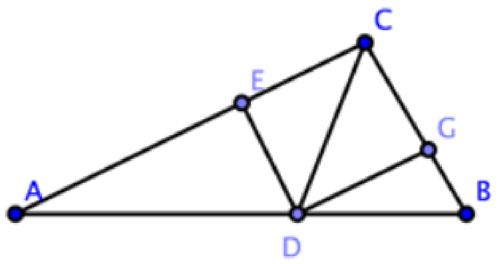
$$\frac{DA}{AB} = \frac{DB}{BC}$$

$\therefore \triangle BAD \sim \triangle CBD$ (SAS)

$\therefore \angle ADB = \angle BDC$

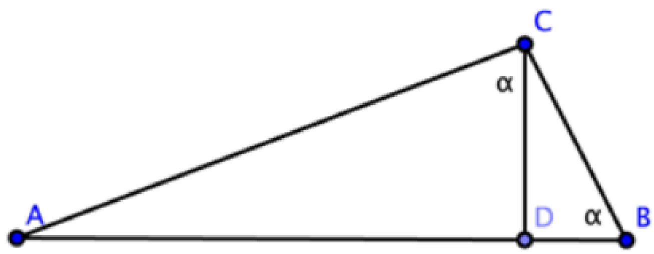
DB bisects $\angle ADC$

10



$$\triangle AED \sim \triangle ACB \text{ AAA}$$
$$\therefore \frac{AE}{AC} = \frac{ED}{CB}$$
$$\therefore AE \times CB = ED \times AC$$
$$\therefore (AC - EC) \times CB = ED \times AC$$
$$\therefore AC \times CB - EC \times CB = ED \times AC$$
$$\therefore AC \times CB = EC \times CB + ED \times AC$$
$$\text{But} EC = ED$$
$$\therefore AC \times CB = ED(CB + AC)$$
$$\therefore \frac{1}{ED} = \frac{CB + AC}{AC \times CB}$$
$$= \frac{1}{AC} + \frac{1}{CB}$$

11



a $\triangle ABC \sim \triangle ACD \sim \triangle CBD$ (AAA)

b We have from $\triangle ABC$ and $\triangle ACD$

$$\frac{AC}{AD} = \frac{AB}{AC}$$
$$\therefore AC^2 = AD \times AB \ldots (1)$$

We have from $\triangle ABC$ and $\triangle CBD$

$$\frac{CB}{BA} = \frac{BD}{CB}$$
$$\therefore CB^2 = BA \times BD \ldots (2)$$

Add (1) and (2)

$$AC^2 + CB^2 = AD \times AB + AB \times BD$$
$$= AB(AD + DB)$$
$$= AB^2$$